PUMPING TEST
for Groundwater Aquifers

Analysis and Evaluation

By: Eng. Deeb Abdel-Ghafour

Ramallah
December, 2005
Hydrogeologists try to determine the most reliable values for the hydraulic characteristics of the geological formations.

This course was designed to present the theory behind groundwater flow to a pumping well, and to illustrate the practical development of pumping test solutions.

This course introduces the basic equations of groundwater flow, the analytical techniques that have been developed to solve these equations, and their practical implementation in pumping Test.

Attendees will gain a more understanding about the analysis of pumping tests and the determination of aquifer hydraulic properties.
Course Objectives

- How to analyze pumping test data
- Hydrogeologic properties of aquifers and their significance
- About aquifer properties and conditions as they relate to pumping test.
- Skills for optimizing and planning your pumping test project
- When to apply appropriate Analytical Techniques (Type Curves) for your pumping test
- How to effectively apply AQTESOLV to your projects.
First Session

- Overview of Aquifer Properties and Conditions
- Principles of Pumping Test
- Aquifer Boundaries
- Overview of Aquifer Test
- Equations for Flow to a Pumping Well
Second Session

- Methods of Pumping Test
- Pumping Test Analysis
- Well Performance Tests
- Recovery Test
The West Bank includes three primarily groundwater basins:

- Eastern Basin
- Northeastern Basin
- Western Basin

More than 95% of Palestinian water supply systems for domestic and agricultural use come from groundwater, either from wells or springs.
Main Problems Facing Water Resources

**REASONS**

- Groundwater over-pumping (uncontrolled pumping) from production wells.
- Cluster of wells in specific areas (Drilling of illegal wells)
- Drought conditions that minimize aquifer Recharge

*Water-level decline*

Today's Water Levels

Year 2020
Important Terminology

**Well yield:** is a measure how much water can be withdrawn from the well over a period of time and measured in m$^3$/hr or m$^3$/day.

**Specific capacity:** is referring to whether the well will provide an adequate water supply. Specific capacity is calculated by dividing pumping rate over drawdown (Q/S).

**Static water level:** is the level of water in the well when no water is being taken out.

**Dynamic Water level:** is the level when water is being drawn from the well. The cone of depression occurs during pumping when water flows from all directions toward the pump.
A cone of depression expanding beneath a riverbed creates a hydraulic gradient between the aquifer and river. This can result in induced recharge to the aquifer from the river.
Aquifer Terminology:

Aquiclude:
A water-bearing layer of rock or sediment that is incapable of transmitting water.

Aquifer:
A water-bearing layer of rock or sediment capable of transmitting significant quantities of water.

Aquitard:
A water-bearing layer of rock or sediment that transmits small quantities of water in relation to Aquifer.
<table>
<thead>
<tr>
<th><strong>Terminology Continue</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Confined Aquifer:</strong></td>
</tr>
<tr>
<td>An aquifer whose upper and lower boundaries are defined by aquicludes.</td>
</tr>
<tr>
<td><strong>Drawdown:</strong></td>
</tr>
<tr>
<td>the amount of water level decline in a well due to pumping. Usually measured relative to static (non-pumping) conditions.</td>
</tr>
<tr>
<td><strong>Unconfined Aquifer:</strong></td>
</tr>
<tr>
<td>An aquifer in which the water table forms the upper boundary.</td>
</tr>
<tr>
<td><strong>Potentiometric Surface:</strong></td>
</tr>
<tr>
<td>an imaginary surface to which water would rise in wells from a given point in confined aquifer. The water table is a particular potentiometric surface for unconfined aquifers.</td>
</tr>
</tbody>
</table>
Hydraulic Properties

Hydraulic Conductivity (K)

- This property is a constant of proportionality that describes fluid flow through a porous media. \( K \) is a function of the permeability of the media and of the physical properties of the fluid, and is generally considered appropriate for evaluation of aquifer properties.

Darcy’s Law: states that the rate of flow through a porous medium is proportional to the loss of head, and inversely proportional to the length of the flow path, or

\[
v = K \left( \frac{dh}{dl} \right)
\]

where,
\[v = \frac{Q}{A},\] which is the specific discharge, or Darcy velocity, (length/time).
\[Q = \text{the volume rate of flow (length}^3/\text{time}).\]
\[A = \text{the cross sectional area normal to flow direction (length}^2).\]
\[dh/dl = \text{aquifer hydraulic gradient (dimensionless) and,}\]
\[K = \text{hydraulic conductivity (length/time)}.\]
Hydraulic Properties

Specific Storage (Ss)
Volume of water released from storage from a unit volume of aquifer per unit decline in hydraulic head. \([1/L]\)

Specific Yield (Sy) is the volume of water yield by gravity drainage to the volume of the aquifer. The specific yield is dimensionless and typically ranges from 0.01 to 0.3.

Storativity (S)
The storativity of a confined aquifer is the volume of water released from storage per unit surface area per unit change in head. The storativity is dimensionless and typically ranges from \(5 \times 10^{-5}\) to \(5 \times 10^{-3}\). \[S = Ss \times B\]

Transmissivity (T)
The product of hydraulic conductivity and Aquifer
The following table shows representative values of hydraulic conductivity for various unconsolidated sedimentary materials, and sedimentary rocks:

<table>
<thead>
<tr>
<th>Material</th>
<th>Hydraulic Conductivity (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel</td>
<td>$3 \times 10^{-4}$ to $3 \times 10^{-2}$</td>
</tr>
<tr>
<td>Coarse sand</td>
<td>$9 \times 10^{-7}$ to $6 \times 10^{-3}$</td>
</tr>
<tr>
<td>Medium sand</td>
<td>$9 \times 10^{-7}$ to $5 \times 10^{-4}$</td>
</tr>
<tr>
<td>Fine sand</td>
<td>$2 \times 10^{-7}$ to $2 \times 10^{-4}$</td>
</tr>
<tr>
<td>Clay</td>
<td>$1 \times 10^{-11}$ to $4.7 \times 10^{-9}$</td>
</tr>
</tbody>
</table>
# Sedimentary Rocks

<table>
<thead>
<tr>
<th>Rock Type</th>
<th>Hydraulic Conductivity (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Karst and reef limestone</td>
<td>1x10^{-6} to 2x10^{-2}</td>
</tr>
<tr>
<td>Limestone, dolomite</td>
<td>1x10^{-9} to 6x10^{-6}</td>
</tr>
<tr>
<td>Sandstone</td>
<td>3x10^{-10} to 6x10^{-6}</td>
</tr>
<tr>
<td>Shale</td>
<td>1x10^{-13} to 2x10^{-9}</td>
</tr>
</tbody>
</table>
Why we should performing AQUIFER TEST?

- Principal part in many projects and studies dealing with groundwater exploitation, protection and remediation.

- Aim to regulate and optimize the extraction without adversely impacting aquifer systems.

- Pumping test gives the best information on the drawdown level, flow rates and unforeseen factors generated upon pumping.
ASM Flowchart

Aquifer System Management

Identifying Data Needs
Monitoring
Sampling
Laboratory Analysis (WQ)

Actions
Reporting
Data Analysis
Data Handling

Numerical (Modeling)
Analytical (Aquifer Test)
Statistical
The principle of a pumping test involves applying a stress to an aquifer by extracting groundwater from a pumping well and measuring the aquifer response to that stress by monitoring drawdown as a function of time. These measurements are then incorporated into an appropriate well-flow equation to calculate the hydraulic parameters of the aquifer.

It can be applied by Single-Well or Multi-Wells (observations).
Pumping well with observation wells in unconfined aquifer
Pumping Test in the Field
Pumping tests are carried out to determine:

- How much groundwater can be extracted from a well based on long-term yield, and well efficiency.
- The hydraulic properties of an aquifer or aquifers.
- Spatial effects of pumping on the aquifer.
- Determine the suitable depth of pump.
- Information on water quality and its variability with time.
Before you start (Design Considerations)

There are several things should be considered before starting a pumping test:

- Literature review for any previous reports, tests and documents that may include data or information regarding geologic and hydrogeologic systems or any conducted test for the proposed area.
- Site reconnaissance to identify wells status and geologic features.
Before you start....Con’t

- Pumping tests should be carried out within the range of proposed or designed rate (for new wells, it should be based on the results of Step drawdown Test).

- Avoid influences such as the pumping of nearby wells shortly before the test.
Before you start….Con’t

- Determine the nearby wells that will be used during the test if it’s likely they will be affected, this well depends on Radius of Influence. The following equation can be used to determine the radius of influence ($R_o$):

$$R_o = \left(\frac{2.25 \times T \times t}{S}\right)^{1/2}$$

This equation can be applied for a pumping well in a confined aquifer.
Before you start….

- Pumping tests should be carried out with open-end discharge pipe in order to avoid back flow phenomena (i.e. $P_p = P_{atm}$).

- Make sure water discharged during the test does not interfere with shallow aquifer tests (Jericho Area).

- Measure groundwater levels in both the pumping test well and nearby wells before 24 hours of start pumping.
Before you start.....Con’t

- Determine the reference point of water level measurement in the well.
- Determine number, location and depth of observation wells (if any).
Equipment Requirements

- **Flow Meter**: flow meter is recommended for most moderates to high flow-rate applications. Others means of gauging flow such as containers could be used for low-flow-rate applications.

- **Water level Indicator**: To be used for measuring static and dynamic water levels such as M-Scope or Data Logger. Water level data should be recorded on aquifer test data sheet.

- **Stop watch**: The project team must have an accurate wrist watch or stop watch. All watches must be synchronized prior to starting pumping test.

- **Personal Requirements**: Most of pumping tests will initially require a minimum of three qualified people. More staff is generally required for long-term constant rate tests with observation wells.
Measuring Water Level by M-Scope
Measuring Pumping Rate by Flow Meter
The measurements to be taken

Water levels measurements for pumping well could be taken as the following:

<table>
<thead>
<tr>
<th>Time since start of pumping (minutes)</th>
<th>Time intervals (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5</td>
<td>0.5</td>
</tr>
<tr>
<td>5-60</td>
<td>5</td>
</tr>
<tr>
<td>60-120</td>
<td>20</td>
</tr>
<tr>
<td>120- shutdown the pump</td>
<td>60</td>
</tr>
</tbody>
</table>
The measurements to be taken

Similarly, for observation wells, water level measurement can be taken as the following:

<table>
<thead>
<tr>
<th>Time since start of pumping (minutes)</th>
<th>Time intervals (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5</td>
<td>30 sec</td>
</tr>
<tr>
<td>5-15</td>
<td>1</td>
</tr>
<tr>
<td>15-50</td>
<td>5</td>
</tr>
<tr>
<td>50-100</td>
<td>10</td>
</tr>
<tr>
<td>100-300</td>
<td>30</td>
</tr>
<tr>
<td>5-48 hr</td>
<td>60</td>
</tr>
<tr>
<td>48 hr- shutdown the pump</td>
<td>480 (8 hr)</td>
</tr>
</tbody>
</table>
The measurements to be taken

After the pump has been shut down, the water levels in the well will start to rise again. These rises can be measured in what is known as recovery test.

If the pumping rate was not constant throughout the pumping test, recovery-test data are more reliable than drawdown data because the water table recovers at a constant rate.

Measurements of recovery shall continue until the aquifer has recovered to within 95% of its pre-pumping water level.
Measurements of well discharge rate

Amongst the arrangements to be made for pumping test is a discharge rate control. This must be kept constant throughout the test and measured at least once every hour, and any necessary adjustments shall be made to keep it constant.
Duration of pumping test

- It’s difficult to determine how many hours that pumping test required because period of pumping depends on the type and natural materials of the aquifer. In general, pumping test is still until pseudo-steady state flow is attained or low fluctuation in dynamic water is occur.

- In some tests, steady state occurs a few hours after pumping, in others, they never occur. However, **24-72 hours** testing is enough to produce diagnostic data and to enable the remaining wells for testing.

- Tests taking longer than 24 hours may be required for large takes, such as community supplies, or situations where it may take longer to determine effects.
Well Testing Stages

- Surging
- Step Drawdown Test
- Recovery Test
- Constant Rate Test
- Recovery Test

Time

Drawdown

Q1, Q2, Q3, Q4, Q5
## PUMPING TEST DATA SHEET

<table>
<thead>
<tr>
<th>Date</th>
<th>Actual time</th>
<th>Elapsed time (t) (min)</th>
<th>Depth to water table (m)</th>
<th>Draw Down (m)</th>
<th>Q (m³/hr)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Recovery Data Sheet

- **Project:**
- **Date:**
- **Sheet:**
- **Name of abstraction well:**
- **Distance from pumped well (m):**
- **Discharge rate during pumping (m³/hr):**
- **S.W.L.:**
- **Remarks:**

<table>
<thead>
<tr>
<th>Actual time</th>
<th>Time (t) since pumping began (min)</th>
<th>Depth to water level (m)</th>
<th>Discharge rate (m³/h)</th>
<th>Drawdown (m)</th>
<th>t/r²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Data Sheet for Recovery**
Location of Observation Wells

- The distance from pumped well should be at Logarithmic Spacing.
- Not too close to pumping well: $\geq 5\text{m}$ or more.
- Located on line parallel to any boundary
- Located on orthogonal line to identify any boundary.
Schematic Array for a test well

Pumping Well

Observation Wells

200m

50m

25m

100m
Data Plots Interpretation
Specific boundary conditions

When field data curves of drawdown versus time deviated from theoretical curves of the main types of aquifer, the deviation is usually due to specific boundary conditions (e.g. partial penetration well, well-bore storage, recharge boundary, or impermeable boundary). These specific conditions may occur individually or in combination.

1- Partial penetration Effect

With partial penetration well, the condition of horizontal flow is not satisfied, at least not in the vicinity of the well. Vertical flow components are inducing extra head losses in the well.
2- well-bore storage

If a pumping test is conducted in large-diameter well, the data will be affected by the well-bore storage in the pumped well. At early pumping time, data will deviate from the theoretical curve.

3- Recharge and Impermeable Boundaries

Recharge or impermeable boundaries can also affect the theoretical curves of all aquifer types. The field data curve then begins to deviate from the theoretical curve up to stabilization in Recharge Case. Impermeable boundaries have the opposite effect on the drawdown. If the cone of depression reaches such a boundary, the drawdown will double.
Theoretical Curve for Confined Aquifer
Theoretical Curve for Unconfined aquifer.
Partial Penetration effect on confined aquifer, the dashed curve is the theoretical time–drawdown data.
The effect of well-bore storage on theoretical time-drawdown data of observation well. The dashed curve is a part of theoretical time–drawdown data.
The effect of a recharge boundary on theoretical confined aquifer. The dashed curve is part of theoretical time – drawdown data.
The effect of impermeable boundary on theoretical confined aquifer. The dashed curve is part of theoretical time–drawdown data.
Data Analysis Methods

**Pumping Test Solution Methods:** (To estimate aquifer properties include single- and multi-well designs).

- Theis (Confined)
- Cooper-Jacob (Time-Drawdown) (Confined)
- Cooper-Jacob (Distance-Drawdown) (Confined)
- Hantush and Jacob (Leaky-Confined)
- Neuman (Unconfined)
- Moench (Unconfined/Partially Penetrating Well)
- Moench (Fracture Flow)

**Step Test Solution Methods:** (are used to determine well performance and efficiency)

- Theis (Confined)
- Cooper-Jacob (Confined)
- Recovery Test (are frequently conducted after pumping is stopped to estimate aquifer properties)
- Theis-Jacob (Theis)
Data Analysis Methods

Slug Test Solution Methods: (are generally conducted as single-well tests)

- Hvorslev
- Bouwer-Rice
- Well Loss Solution Methods
- Hantush-Bierschenk

Other Solution Methods:
- Specific Well Capacity
- Drawdown versus Distance
Theory Background & Mathematical Equations for Water Wells
For confined aquifers, Steady State Flow

from Darcy Law, the flow of water through a circular section of aquifer to well is describe as:

\[ Q = 2\pi r kb \frac{dh}{dr} \]

Since \( T = Kb \)

\[ Q = 2\pi r T \frac{dh}{dr} \]

Rearrange equation as follow:

\[ dh = \left(\frac{Q}{2\pi T}\right) \frac{dr}{r} \]

With two observation wells, and by integrations

\[ \frac{Q}{2\pi T} \int_{r_1}^{r_2} \frac{1}{r} dr = h_1 \int_{h_2}^{h_1} dh \]
Gives $\left( \frac{Q}{2\pi T} \right) \ln \left( \frac{r_2}{r_1} \right) = h_2 - h_1$

In term of drawdown ......

$\left( \frac{Q}{2\pi T} \right) \ln \left( \frac{r_2}{r_1} \right) = s_2 - s_1$

Arrange the equation yields

$T = \frac{kb}{\left( \frac{Q}{2\pi (s_2 - s_1)} \right)} \ln \left( \frac{r_2}{r_1} \right)$

Which is Known as **Thiem** equation
The previous equation can be integrated with the following boundary conditions:

1. At distance $r_w$ (well radius) the head in a well is $h_w$
2. At distance $R$ from well (Radius of influence), the head is $H$ (which is the undisturbed head and equal to initial head before pumping)

So, the equation can be written as:

$$H - h_w = s_w = \left(\frac{Q}{2\pi T}\right) \ln\left(\frac{R}{r_w}\right)$$
For Unconfined aquifers, Steady State Flow
Based on the Dupuit and Forchheimer assumptions:

1. Flow lines are assumed to be horizontal and parallel to impermeable layer
2. The hydraulic gradient of flow is equal to the slope of water. (slope very small)

Radial flow in unconfined aquifer is described by:

\[
Q = (2\pi rh)k(\frac{dh}{dr})
\]

\[
\frac{dh}{dr} = \left(\frac{Q}{2\pi K}\right)\frac{dr}{r}
\]
By integrations

\[ \frac{Q}{2\pi k} \int_{r_1}^{r_2} \frac{dr}{r} = h_1 \int_{h_1}^{h_2} h \, dh \]

Gives

\[ \frac{Q}{2\pi k} \ln \left( \frac{r_2}{r_1} \right) = (h_2^2 - h_1^2) \]

Rearrange equation yield

\[ K = \left( \frac{Q}{2\pi (h_2^2 - h_1^2)} \right) \ln \left( \frac{r_2}{r_1} \right) \]
For confined aquifers, Transient Flow
1. **Theis Method** (Curve Matching Method)

Theis (1935) solved the non-equilibrium flow equations in radial coordinates as follows:

\[ s = \frac{Q}{4\pi T} \int_u^\infty \left(\frac{e^{-u}}{u}\right) du \]

Where the dimensionless parameter (or dummy variable) \( u \) is given as:

\[ u = \frac{r^2 S}{4Tt} \]

Where...

- \( s \) = drawdown (L; m or ft)
- \( r \) = is the distance from pumping well where \( s \) is recorded (L).
- \( S \) = Storage coefficient
- \( t \) = is the time since the beginning of pumping (minutes)
- \( T \) = Transmissivity (L²/t)
1. Theis Method (Curve Matching Method)

For the specific definition of $u$ given above, the integral is known as the **Well Function** $W(u)$, and can be represented by an infinite Taylor series of the following form:

$$W(u) = -0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + \ldots$$

Using this function, the equation becomes:

$$s = \left( \frac{Q}{4\pi T} \right) W(u)$$
General Assumption and Limitations for Theis Method

- Prior to pumping, the potentiometric surface is approximately horizontal (No slope).
- The aquifer is confined and has an "apparent" infinite extent.
- The aquifer is homogeneous, isotropic, of uniform thickness over the area influenced by pumping.
- The well is pumped at a constant rate.
- The well is fully penetrating.
- Water removed from storage is discharged instantaneously with decline in head.
- The well diameter is small so that well storage is negligible.
The data required for the Theis solution are:

- Drawdown vs. time data at an observation well
- Distance from the pumping well to the observation well
- Pumping rate of the well.
The procedure for finding parameters by Theis Method

- Theoretical curve $W(u)$ versus $1/u$ is plotted on a log-log paper.
- The field measurements are similarly plotted on a log-log plot with $(t)$ along the x-axis and $(s)$ along the y-axis.
- The data analysis is done by matching the observed data to the type curve.
- From the match point, determine the values for $W(u)$, $1/u$, $s$, and $t$.
- Use the previous equations to determine $T$ and $S$. 
Type Curve (used for Curve Matching Method, the Theis Method)

- $W(u)$ vs. $1/u$ graph
- $W(u)$ values range from 0.1 to 10
- $1/u$ values range from 1 to 10,000
## APPENDIX 1
Values of the function $W(u)$ for various values of $u$

<table>
<thead>
<tr>
<th>$u$</th>
<th>$W(u)$</th>
<th>$u$</th>
<th>$W(u)$</th>
<th>$u$</th>
<th>$W(u)$</th>
<th>$u$</th>
<th>$W(u)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 10^{-10}$</td>
<td>22.45</td>
<td>$7 \times 10^{-8}$</td>
<td>15.90</td>
<td>$4 \times 10^{-5}$</td>
<td>9.55</td>
<td>$1 \times 10^{-2}$</td>
<td>4.04</td>
</tr>
<tr>
<td>2</td>
<td>21.76</td>
<td>8</td>
<td>15.76</td>
<td>5</td>
<td>9.33</td>
<td>2</td>
<td>3.35</td>
</tr>
<tr>
<td>3</td>
<td>21.35</td>
<td>9</td>
<td>15.65</td>
<td>6</td>
<td>9.14</td>
<td>3</td>
<td>2.96</td>
</tr>
<tr>
<td>4</td>
<td>21.06</td>
<td>$1 \times 10^{-7}$</td>
<td>15.54</td>
<td>7</td>
<td>8.99</td>
<td>4</td>
<td>2.68</td>
</tr>
<tr>
<td>5</td>
<td>20.84</td>
<td>2</td>
<td>14.85</td>
<td>8</td>
<td>8.86</td>
<td>5</td>
<td>2.47</td>
</tr>
<tr>
<td>6</td>
<td>20.66</td>
<td>3</td>
<td>14.44</td>
<td>9</td>
<td>8.74</td>
<td>6</td>
<td>2.30</td>
</tr>
<tr>
<td>7</td>
<td>20.50</td>
<td>4</td>
<td>14.15</td>
<td>$1 \times 10^{-4}$</td>
<td>8.63</td>
<td>7</td>
<td>2.15</td>
</tr>
<tr>
<td>8</td>
<td>20.37</td>
<td>5</td>
<td>13.93</td>
<td>2</td>
<td>7.94</td>
<td>8</td>
<td>2.03</td>
</tr>
<tr>
<td>9</td>
<td>20.25</td>
<td>6</td>
<td>13.75</td>
<td>3</td>
<td>7.53</td>
<td>9</td>
<td>1.92</td>
</tr>
<tr>
<td>$1 \times 10^{-9}$</td>
<td>20.15</td>
<td>7</td>
<td>13.60</td>
<td>4</td>
<td>7.25</td>
<td>$1 \times 10^{-1}$</td>
<td>1.823</td>
</tr>
<tr>
<td>2</td>
<td>19.45</td>
<td>8</td>
<td>13.46</td>
<td>5</td>
<td>7.02</td>
<td>2</td>
<td>1.223</td>
</tr>
<tr>
<td>3</td>
<td>19.05</td>
<td>9</td>
<td>13.34</td>
<td>6</td>
<td>6.84</td>
<td>3</td>
<td>0.906</td>
</tr>
<tr>
<td>4</td>
<td>18.76</td>
<td>$1 \times 10^{-6}$</td>
<td>13.24</td>
<td>7</td>
<td>6.69</td>
<td>4</td>
<td>0.702</td>
</tr>
<tr>
<td>5</td>
<td>18.54</td>
<td>2</td>
<td>12.55</td>
<td>8</td>
<td>6.55</td>
<td>5</td>
<td>0.560</td>
</tr>
<tr>
<td>6</td>
<td>18.35</td>
<td>3</td>
<td>12.14</td>
<td>9</td>
<td>6.44</td>
<td>6</td>
<td>0.454</td>
</tr>
<tr>
<td>7</td>
<td>18.20</td>
<td>4</td>
<td>11.85</td>
<td>$1 \times 10^{-3}$</td>
<td>6.33</td>
<td>7</td>
<td>0.374</td>
</tr>
<tr>
<td>8</td>
<td>18.07</td>
<td>5</td>
<td>11.63</td>
<td>2</td>
<td>5.64</td>
<td>8</td>
<td>0.311</td>
</tr>
<tr>
<td>9</td>
<td>17.95</td>
<td>6</td>
<td>11.45</td>
<td>3</td>
<td>5.23</td>
<td>9</td>
<td>0.260</td>
</tr>
<tr>
<td>$1 \times 10^{-8}$</td>
<td>17.84</td>
<td>7</td>
<td>11.29</td>
<td>4</td>
<td>4.95</td>
<td>$1 \times 10^{0}$</td>
<td>0.219</td>
</tr>
<tr>
<td>2</td>
<td>17.75</td>
<td>8</td>
<td>11.16</td>
<td>5</td>
<td>4.73</td>
<td>2</td>
<td>0.049</td>
</tr>
<tr>
<td>3</td>
<td>17.64</td>
<td>9</td>
<td>11.04</td>
<td>6</td>
<td>4.54</td>
<td>3</td>
<td>0.013</td>
</tr>
<tr>
<td>4</td>
<td>16.46</td>
<td>$1 \times 10^{-5}$</td>
<td>10.94</td>
<td>7</td>
<td>4.39</td>
<td>4</td>
<td>0.004</td>
</tr>
<tr>
<td>5</td>
<td>16.23</td>
<td>2</td>
<td>10.24</td>
<td>8</td>
<td>4.26</td>
<td>5</td>
<td>0.001</td>
</tr>
<tr>
<td>6</td>
<td>16.05</td>
<td>3</td>
<td>9.84</td>
<td>9</td>
<td>4.14</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Matching point
Figure 16.31. Illustration of the Theis curve-matching technique.
2. Cooper-Jacob Method (Time-Drawdown, Confined)

This method was modified based on Theis equation

For small values of \( u \) (small value of (r) or large value of pumping time \( t \))

The \( u, u^2, u^3, \ldots \) terms can be ignored and \( W(u) \) can be approximated as

\[
W(u) = -0.5772 - \ln u
\]

\[
s = \left( \frac{Q}{4\pi T} \right) \cdot W(u) = \left( \frac{Q}{4\pi T} \right) \cdot (-0.5772 - \ln r^2S/4Tt)
\]

rearrange eq

\[
s = \left( \frac{2.303Q}{4\pi T} \right) \cdot \log_{10} \left( \frac{2.25Tt}{r^2S} \right)
\]

Jacob method is valid for \( u \leq 0.01 \).
- The above equation plots as a straight line on semi-logarithmic paper.
- Time is plotted along the logarithmic x-axis, while drawdown is plotted along the linear y-axis.
For the (t-s) method, Transmissivity and storativity are estimated as follow:

\[ T = \frac{2.3Q}{4\pi \Delta s} \text{ per one logarithmic cycle} \]

From Eq. \[ s = \frac{2.3Q}{4\pi T} \log_{10} \left( \frac{2.25Tt}{r^2 S} \right) \]

When \( s=0 \) → \[ \frac{2.25Tt}{r^2 S} = 1 \]

therefore,

\[ S = \frac{2.25Tt_0}{r^2} \]
\[ T = \frac{2.303Q}{4\pi s} \]
\[ = \frac{2.303 \times 40 \times 86.4}{4 \times \pi \times 1.95} \]
\[ = 325 \text{ m/day} \]

\[ S = \frac{2.25Tl_0}{r^2} \]
\[ = \frac{2.25 \times 325 \times 450}{1440 \times 100 \times 100} \]
\[ = 2.29\% \]

\( Q = 40 \text{ l/s} \)
\( r = 100 \text{ m} \)
\( \Delta s = 1.95 \text{ m} \)

Note 86.4 and 1440 are unit conversion factors.
Problems that may be encountered during analysis

The ideal line

Graph showing s (m) versus time (min) with three values of Q: Q=200, Q=100, Q=220.
Remember!

Cooper-Jacob Method (Time-Drawdown) is highly recommended for pumping tests with single well (conditioned by t?).
3. Cooper-Jacob Method (Distance-Drawdown, Confined)

- If simultaneous observations are made of drawdown in three or more observation wells, the observation well distance is plotted along the logarithmic x-axes, and drawdown is plotted along the linear y-axes.

- For the Distance-Drawdown method, transmissivity and storativity are calculated as follows:

\[ T = \frac{2.3Q}{4\pi \Delta s} \]  

per one logarithmic cycle

When \( s=0 \)  \[ \rightarrow \quad 2.25Tt/r^2S=1 \]

therefore, \[ S = \frac{2.25Tt}{r_o^2} \]

Where, delta \( (s) \) is the change in drawdown over one logarithmic cycle, \( (r_o) \) is the distance defined by the intercept of the straight-line fit of the data and zero-drawdown axis, and \( (t) \) is the time to which the set of drawdown data correspond.
Straight line plot of Cooper-Jacob Method (Distance-Drawdown, Confined)

\[ r_0 = 460 \text{ ft} \]

\[ \Delta(h_0 - h) = 8.8 \text{ ft} \]
4. Hantush and Jacob Method for Leaky (Semi) Confined Aquifer
The Hantush and Jacob (1955) solution for leaky aquifer presents the following equations:

\[ s = \left( \frac{Q}{4\pi T} \right) W [u, r/B] \rightarrow T = \left( \frac{Q}{4nT} \right) W [u, r/B] \]

where \( u = \frac{r^2S}{4Tt} \rightarrow S = 4Tut/r^2 \)

\( W [u, r/B] \): is the well function for leaky confined aquifer
\( B \): Is the leakage factor given as

\[ K' = \frac{[Tb'(r/B)^2]}{r^2} \]

\( K' \) is the vertical hydraulic conductivity of confining bed (aquitard) (L/t)
\( b' \) is the thickness of aquitard (L)
\( B \) is the leakage factor, \( (Tb'/K')^{1/2} \)
Hantush Method...

- A log/log plot of the relationship $W(u, r/B)$ along the y axis versus $1/u$ along the x axis is used as the type curve as with the Theis method.

- The field measurements are plotted as $t$ along the x-axis and $s$ along the y-axis. The data analysis is done by curve matching.
log/log plot of Hantush Method
The Hantush and Jacob solution has the following assumptions:

- The aquifer is leaky and has an "apparent" infinite extent.
- The aquifer and the confining layer are homogeneous, isotropic, and of uniform thickness over the area influenced by pumping.
- The potentiometric surface was horizontal prior to pumping.
- The well is pumped at a constant rate.
- The well is fully penetrating.
- Water removed from storage is discharged instantaneously with decline in head.
- The well diameter is small so that well storage is negligible.
- Leakage through the aquitard layer is vertical.
Hantush Method....

The data requirements for the Hantush solution are:

- drawdown vs. time data at an observation well
- distance from the pumping well to the observation well
- pumping well rate.
5. Neuman (Unconfined)

In general all previous techniques of confined aquifer can be used for unconfined aquifer, BUT an adjustment should be done for drawdown as follow:

\[ s' = s - \left( \frac{s^2}{2h} \right) \]

Where

- \( s' \) is the adjusted drawdown
- \( h \) is the initial saturated thickness of aquifer

Neuman introduce the following flow equation for unconfined aquifer:

\[ S = \left( \frac{Q}{4\pi T} \right) W(u_a, u_b, \Gamma) \]
Where
$W(u_a, u_b, \Gamma)$ is the well function of water table and
$S = (4T u_a t)/r^2$.....for early drawdown data
$S_y = (4T u_b t)/r^2$.....for later drawdown data
$\Gamma = (r^2 K_v)/(b^2 K_h)$

Where,
$S$ is the storativity
$S_y$ is the specific yield
$R$ radial distance from pumping well
$b$ is the initial saturated thickness of aquifer
$K_v$ is horizontal hydraulic conductivity
$K_h$ is horizontal hydraulic conductivity
Neuman (Unconfined)

- Two sets of type curves are used and plotted on log-log paper (Theoretical curve $W(u_a, u_b, \Gamma)$ versus $1/u$).
- Superpose the early (t-s) data on Type-A curve.
- The data analysis is done by matching the observed data to the type curve.
- From the match point of Type-A curve, determine the values for $W(u_a, \Gamma), 1/u_a, s, t, \Gamma$.
- Use the previous equations to determine T and S.
- The latest s-t data are then superposed on Type-B Curve for the $\Gamma$-values of previously matched Type-A curve, from the match point of Type-b curve, determine the values for $W(u_b, \Gamma), 1/u_b, s, t$.
- By using the previous equations, the T and S can be determined.
Type Curves for Unconfined Aquifers

\[ \Gamma = \frac{r^2 K_f}{K_h b^2} \]
Step drawdown test

- Step drawdown test developed to assess the Well Performance (Well losses due turbulent flow).
- The well is pumped at several successively higher pumping rates and (s) for each rate (step) is recorded with time.
- At least 5 pumping steps are needed, each step lasting from 1 to 2 hours.
- Step drawdown test is used to determine the Optimum Pumping Rate.
- Step drawdown test can be used to determine T and S from each step.
Step drawdown test

- Jacob suggest that the total drawdown in a well can represented by:

\[ s_T = s_a + s_w = BQ + CQ^2 \]

Where...

- \( s_T \) is the total drawdown (L)
- \( s_a = BQ \) is part of drawdown due to Aquifer losses (as laminar Term)
- \( s_w = CQ^2 \) is part of drawdown due to well losses (as Turbulent Term)
- \( Q \) = Pumping Rate (L^3/t)
Dividing the equation by \( Q \) yields:

\[
s/Q = B + CQ
\]

This form is a linear equation in \( s/Q \) and \( Q \), so if \( s/Q \) is plotted against \( Q \), the resultant graph is a straight line with slope \( C \) and intercept \( B \).
Figure 16.15. Values for \( B \) and \( C \) in the step-drawdown equation can be determined from a graph where \( s/Q \) is plotted against \( Q \).

\[
B = 0.0225 \\
C = 3.68 \times 10^{-5}
\]

\[
L_p = \frac{BQ}{BQ + CQ^2} = 69\% \text{ laminar flow}
\]
The relationship between $Q$ and $s/Q$ is given by the equation:

$$s/Q = 0.0013Q + 0.28$$
Another parameter can be computed from step-drawdown test:

\[ L_p = \left( \frac{BQ}{BQ + CQ^2} \right) \times 100 \]

Where

\( L_p \) is the ratio of laminar head losses to the total head losses (this parameter can be considered also as well efficiency)
Well Efficiency is the ratio between theoretical drawdown and the actual drawdown measured in the well expressed as:

\[
\text{Well Efficiency} = \left( \frac{\text{Theoretical Drawdown outside the well}}{\text{Measured Drawdown inside the well}} \right) \times 100\%
\]

- A well efficiency of 70% or more is usually acceptable.
- If a newly developed well has less than 65% efficiency, it should not be accepted.
Drawdown Configuration

- Static Water Level
- Aquifer Water Level
- Pumping Water Level
- Total Drawdown
- Theoretical Drawdown outside the well
Figure 9.31. Theoretical drawdown of a pumped well can be compared with the actual drawdown by extending the straight line on the distance-drawdown diagram to a point where the radius of the well (outer face of the well) is indicated on the horizontal scale.
Specific Well Capacity

Step-drawdown test can be used also to determine Specific Capacity of a well at various discharge rates.

Inverting the terms in this equation \( s/Q = B + CQ \) yields:

\[
Q/s = \frac{1}{B + CQ}
\]

\( Q/s \) is defined as Specific Capacity (L\(^3\)/t/L)

SC is decreased as \( Q \) increased.

SC can be also determined (\( Q/s \)) from the constant rate test. It’s important parameter that can gives indication about future well productivity, degree of well development, …
Transmissivity can be estimated following Driscoll, 1986

\[ T = 1500 \times \frac{Q}{s} \] (unconfined)

\[ T = \frac{Q}{s} \times 2000 \] (confined)

\( T = \text{gpd/ft}; \ Q = \text{gpm}; \ s = \text{ft} \)

Note: this calculation assumes \( t = 1 \) day, \( r = 0.5 \) ft \( T = 30,000 \) gpd/ft; \( S = 0.003 \) for a confined aquifer and 0.075 for unconfined aquifer, errors of less than 7% are reported by Driscoll.

Other Empirical Equations can be used (with units of m-day):

\[ T = 1.81 (\frac{Q}{s})^{0.917} \] fractured carbonate aquifer, El-Naqa (1994)
\[ T = 1.23 (\frac{Q}{s})^{1.05} \] carbonate aquifer, Mace (1997)
\[ T = 0.785 (\frac{Q}{s})^{1.07} \] fractured carbonate aquifer, Fabbri (1997)
\[ T = 0.78 (\frac{Q}{s})^{0.98} \] limestone aquifer, Edwards-Trinity
\[ T = 15.3 (\frac{Q}{s})^{0.67} \] Razack and Huntley, 1991
Optimum Pumping Rate

Determining the Optimum Pumping Rate is based mainly on the well losses and well efficiency, the procedure consists of the following steps:

- For up to ten different Q, find $s_T$ based on the previous equation $s_T=BQ+CQ^2$
- For the same pumping rates, find theoretical (s) through the following equation:
  $$s=(Q/2\pi T)\ln(R/r_w)$$
- Calculate well efficiency for all pumping rates
- Demonstrate graph between efficiencies and pumping rates, and choose the Q value that correspond more than 65% efficiency or more.
Optimum Q

![Graph showing the relationship between Pumping Rate (m³/hr) and Efficiency %]
Recovery Test

When pumping is stopped, water level rise towards it pre-pumping level. The resulting drawdown at any time after pumping stop is algebraic sum of \((s)\) from well and buildup (negative drawdown) from imaginary recharge well.
Residual Drawdown \[ s' = s + s_{rec} \]

The above drawdown components are expressed with Theis equation as:

\[ s' = (+Q/4\pi T)(W(u) + (-Q/4\pi T(W(u'))) \]

\[ s' = (Q/4\pi T)[W(u) - (W(u'))] \]

Where

\[ u = (r^2S)/4Tt \quad \text{and} \quad u' = (r^2S)/4Tt' \]

\( t \) is time since pumping started
\( t' \) is time since pumping stopped

When \( u \) is less than 0.01, Theis eq can be simplified by Jacob and Cooper eq as:
s' = \frac{Q}{4\pi T} \left\{ \ln(2.25\frac{Tt}{r^2S}) - \ln(2.25\frac{T't'}{r'^2S}) \right\}

or

s' = \left( \frac{Q}{4\pi T} \right) \ln(\frac{t}{t'});

or

s' = \left( \frac{2.303Q}{4\pi T} \right) \log(\frac{t}{t'});

\Delta s' = \text{Drawdown per log cycle}
\Delta s' = \left( \frac{2.303Q}{4\pi T} \right);

or

T = \left( \frac{2.303Q}{4\pi \Delta s'} \right);

Note: this method of recovery analysis \((s'-t/t')\) does not allow calculation of \(S\), this is obvious from the absence of \(S\) in basic equations of this method.
\[ T = \frac{2.303Q}{4\pi \Delta s} \]

\[ = \frac{2.303 \times 40 \times 68.4}{4 \times \pi \times 1.81} \]

\[= 349 \text{m}^2/\text{d} \]
Another method for recovery analysis based on Jacob Equation, it based on using $t'$ since pumping stopped.

This method enabled the calculation of Storage Coefficient.

$$s_{rec} = s - s'$$

From Eq...

$$s_{rec} = (2.303Q/4\pi T) \log_{10} \left( \frac{2.25Tt'}{r^2S} \right)$$

When $s_{rec} = 0 \quad \Rightarrow \quad 2.25Tt/r^2S = 1$

therefore,

$S = 2.25Tt/r^2$
Calculated recovery $s_{rev}$ (m)

$t_0' = 1.2 \text{ min}$

$\Delta s_{rev} = 1.66 \text{ m}$

Time since pumping stopped $t'$ (min)
Pumping Test reports

Pumping test reports should include the following:

- A map, showing the location of the investigated site, pumping and observation wells.
- Details on the existed wells (main data).
- Well logs and construction details for all wells.
- Geological cross-section of the study area.
- Tables of field measurements: Drawdown measurements, time of measurement and flow rate (including soft copy).
- The calculations in an abbreviated form, including the values obtained for the aquifer parameters and discussion of their accuracy.
- Recommendations.
- Summary.
Recommended References for further reading:

Training by using AQTESOLV Software
Thank You