2.10 Water Level Maps

Maps of the water table for an unconfined aquifer or of the potentiometric surface of a confined aquifer are basic tools of hydrogeologic interpretation. These maps are two-dimensional representations of three dimensional view of the surface.

The data used to construct water-table and potentiometric-surface maps are water-level elevations as measured in wells. However, not every well is useful for this purpose. Some water-supply wells are open borings in rocks that include both aquifers and confining beds. Other water-supply wells may have more than one well screen, each opposite a different aquifer unit and not one specific aquifer unit, they are not useful in making water-level maps.

In order to make a ground-water level map, one needs water-level readings made in a number of wells, each of which is open only in the aquifer of interest. Since groundwater levels can change with time, all the readings should be made within a short period of time. Some measuring point on each well needs to be surveyed to a common datum so that the water levels can be referenced to a height above the datum; mean sea level is a common datum for this purpose.

If water levels are to be measured in a well that is normally used for water supply, one must make sure that the pump has been shut off long enough for the water level to recover to what is termed the static, or nonpumping, level. Depth to water measurements can be made every few minutes until the water level stops rising.

When making a water-table map, it is ideal if all the wells have an open borehole or a well screen at the depth of the water table. However, wells that are cased or screened below the water table can be used if they don't extend too far below the water table. For wells used to make a potentiometric-surface map, all aquifers above the aquifer of interest should be cased off.

Surface-water features such as springs, ponds, lakes, streams, and rivers can interact with the water table. In addition, the water table is often a subdued reflection of the surface topography. All this must be taken into account when preparing a water-table map. A base map showing the surface topography and the locations of surface-water features should be prepared. The elevations of lakes and ponds can be helpful information. The locations of the wells are then plotted on the base map, and the water-level elevations are noted. The datum for the water level in wells should be the same as the datum for the surface topography. Interpolation of contours between data points is strongly influenced by the surface topography and surface water features. For example, groundwater contours cannot be higher than the surface topography. The depth to groundwater will typically be greater beneath hills than beneath valleys. If a lake is present, the lake surface is flat and the water table beneath it is also flat. Hence, groundwater contours must go around it. The only exception to this rule is when the lake is perched on low-permeability sediments and has a surface elevation above the main water table. Groundwater contours form a V, pointing upstream when they cross a gaining stream. Groundwater contours bend downstream when they cross a losing stream.

Figure 2.12A is a water-table map where there is a gaining stream and a lake that is hydraulically connected with the water table. **Figure 2.12B** is a water table map where there is a perched lake. Water is seeping from the perched lake, so the water table contours bend down-gradient away from the lake.

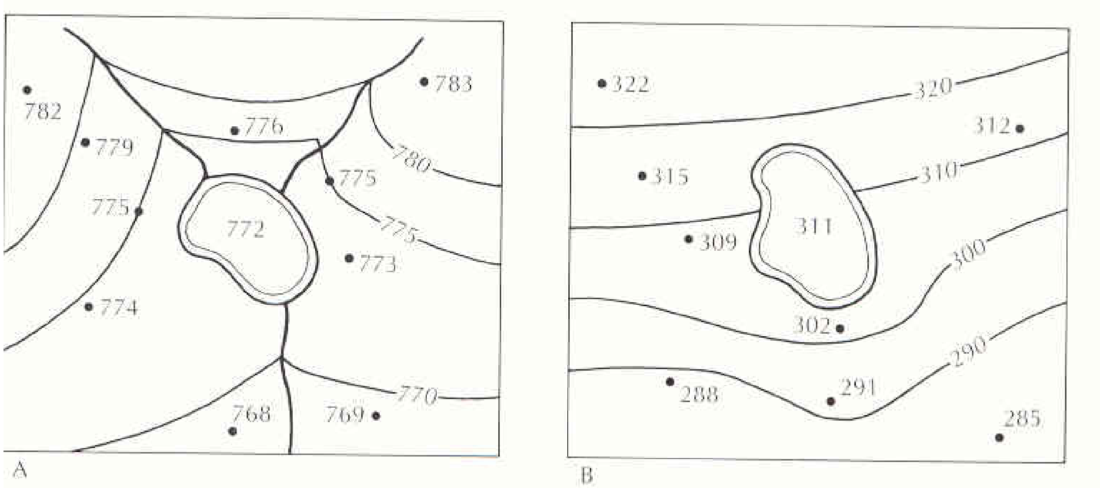


Figure 2.12 Maps showing construction of water table maps in areas with surface-water bodies. **A**. A water table lake with two gaining streams draining into it and one gaining stream draining from it. **B**. A perched lake that, through outseepage, is recharging the water table.

In general, the potentiometric surface of a confined aquifer is not influenced by the surface topography and surface-water features. Because there is no hydraulic connection between a river and a confined aquifer beneath it, potentiometric-surface contours can even be above the land surface. This indicates that if a well were to be drilled at that location, it would flow.

In areas where the water table or potentiometric surface has a shallow gradient, the groundwater contours will be spaced well apart. If the gradient is great, the groundwater contours will be spaced well apart. If the gradient is small, the groundwater contours will be closer together. Groundwater will flow in the general direction that the water table or potentiometric surface is sloping.

2.11 Gradient of Hydraulic Head

The potential energy, or force potential, Φ, of groundwater consists of two parts: the elevation and pressure. It is equal to the product of the acceleration of gravity and the total head, and represents mechanical energy per unit mass:

$$\Phi = g\,h \qquad \textbf{(2.114)}$$

Force potential is a physical quantity. To obtain it one needs only to measure the heads in an aquifer with piezometers and multiply the results by the acceleration of gravity. If a point in an aquifer has a head of 15.1 m and the value of g is 9.81 m/s^2, then Φ is 15.1 x 9.81 = 148.1 m^2/s^2. For practical purposes, as g is usually constant throughout an area, most field problems are solved in terms of hydraulic head, h.

If the value of h is variable in an aquifer, a contour map may be made showing lines of equal values of h (equipotential lines). Such a map is similar to topographic map of the land-surface elevation. In three-dimensional cases, one deals with surfaces of equal value of h (equipotential surfaces)

Figure 2.13 shows equipotential surfaces for a two-dimensional uniform flow field. The equipotential surfaces are vertical, and they form equipotential lines with a horizontal plane. The equipotential field is uniform-that is, the horizontal distance between each equipotential surface is the same. Also shown is a vector known as the gradient of h (grad h). Remember, a vector is a directed line segment, so that the grad h has a magnitude and direction. It is roughly analogous to the maximum slope of the potential field. In the notation of differential calculus,

$$grad\ h = \frac{dh}{ds} \qquad \textbf{(2.115)}$$

where s is the distance parallel to grad h. Grad h has a direction perpendicular to the equipotential lines.

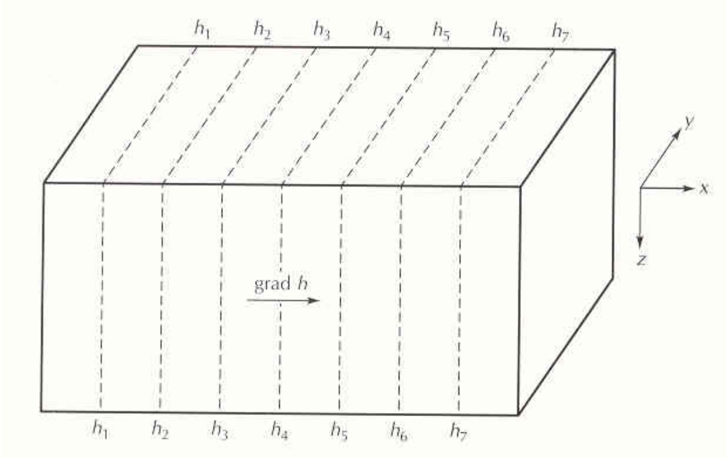


Figure 2.13 Equipotential lines in a three-dimensional flow field and the gradient of h.

If the potential is the same everywhere, it will be manifest in a condition such as a flat water table. In this case, grad h equal zero, since there is no slope to h. There will be no groundwater flow, since grad h must have a positive value before groundwater will move.

2.12 Flow Lines and Flow Nets

A flow line is an imaginary line that traces the path that a particle of groundwater would follow as it flows through an aquifer. Flow lines are helpful for visualizing the movement of groundwater. In an isotropic aquifer, flow lines will cross equipotential lines at right angles. If there is anisotropy in the plane of flow, then the flow lines will cross the equipotential lines at an angle dictated by the degree of anisotropy and the orientation of grade h to the hydraulic conductivity tensor ellipsoid. **Figure 2.14A** shows equipotential lines and flow lines in an isotropic medium and **Figure 2.14B** shows equipotential lines and flow lines in an anisotropic medium. It may be seen that in the isotropic medium the flow lines are parallel to grad h, and in the anisotropic medium they are not.

Laplace equation for steady-flow conditions may be solved by graphical construction of **flow net**, which is a network of equipotential lines and associated flow lines.

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \qquad \textbf{(2.116)}$$

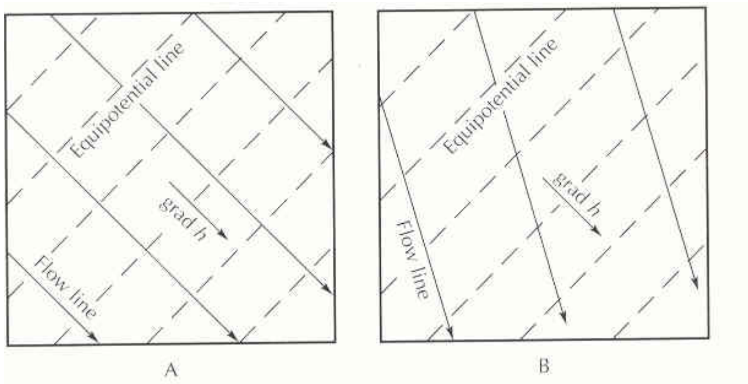


Figure 2.14 Relationship of flow lines to equipotential field and grad h. **A.** Isotropic aquifer. **B.** Anisotropic aquifer

A flow net is especially useful in anisotropic media. However, with certain transformations it can be used with anisotropic aquifers. Cedergren (1989) presents a complete discussion of the construction of flow nets, including those in anisotropic media.

The method of flow-net construction presented here is based on the following assumptions.

1. The aquifer is homogeneous.
2. The aquifer is fully saturated.
3. The aquifer is isotropic.
4. There is no change in the potential field with time.
5. The soil and water are incompressible.
6. Flow is laminar, and Darcy's law is valid.
7. All boundary conditions are known.

There are three types of **boundary conditions** possible. Groundwater cannot pass a **no-flow boundary.** Adjacent flow lines will be parallel to a no-flow boundary, and equipotential lines will intersect it at right angles. Since the head is the same everywhere on a **constant-head boundary**, such a boundary represents an equipotential line. Flow lines will intersect a constant-head boundary at right angles and the adjacent equipotential line will be parallel. For unconfined aquifers, there's also a **water-table boundary.** The water table is neither a flow line nor an equipotential line; rather it is a line where head is known. If there is recharge or discharge across the water table, flow lines will be at an oblique angle to the water table. If there is no recharge across the water table, flow lines can be parallel to it.

A flow net is a family of equipotential lines with sufficient orthogonal flow lines drawn so that a pattern of "square" figures results. Except in cases of the most simple geometry, the figures will not truly be squares. The following steps are followed in the construction of a flow net.

1. Identify the boundary conditions.
2. Make a sketch of the boundaries with the two axes of the drawing having the same scale.
3. Identify the position of known equipotential and flow-line conditions.
4. Draw a trial set of flow lines. The outer flow lines will be parallel to no-flow boundaries. The distance between adjacent flow lines should be the same at all sections of the flow field.
5. Draw a trial set of equipotential lines. Start at one end of the flow field and work toward the other. The equipotential lines should be perpendicular to flow lines. They will be parallel to constant-head boundaries. If there is a water-table boundary, the position of the equipotential line at the water table is based on the elevation of the water table. The equipotential lines should be spaced so that they form areas that are equidimensional. These are not necessarily squares, but should be as square as possible.

6 Erase and redraw the trial flow lines and equipotential lines until the desired flow net of orthogonal equipotential lines and flow lines is obtained.

Figure 2.15 illustrates the construction of a very simple flow net, where the pattern forms exact squares because the flow field is rectangular.

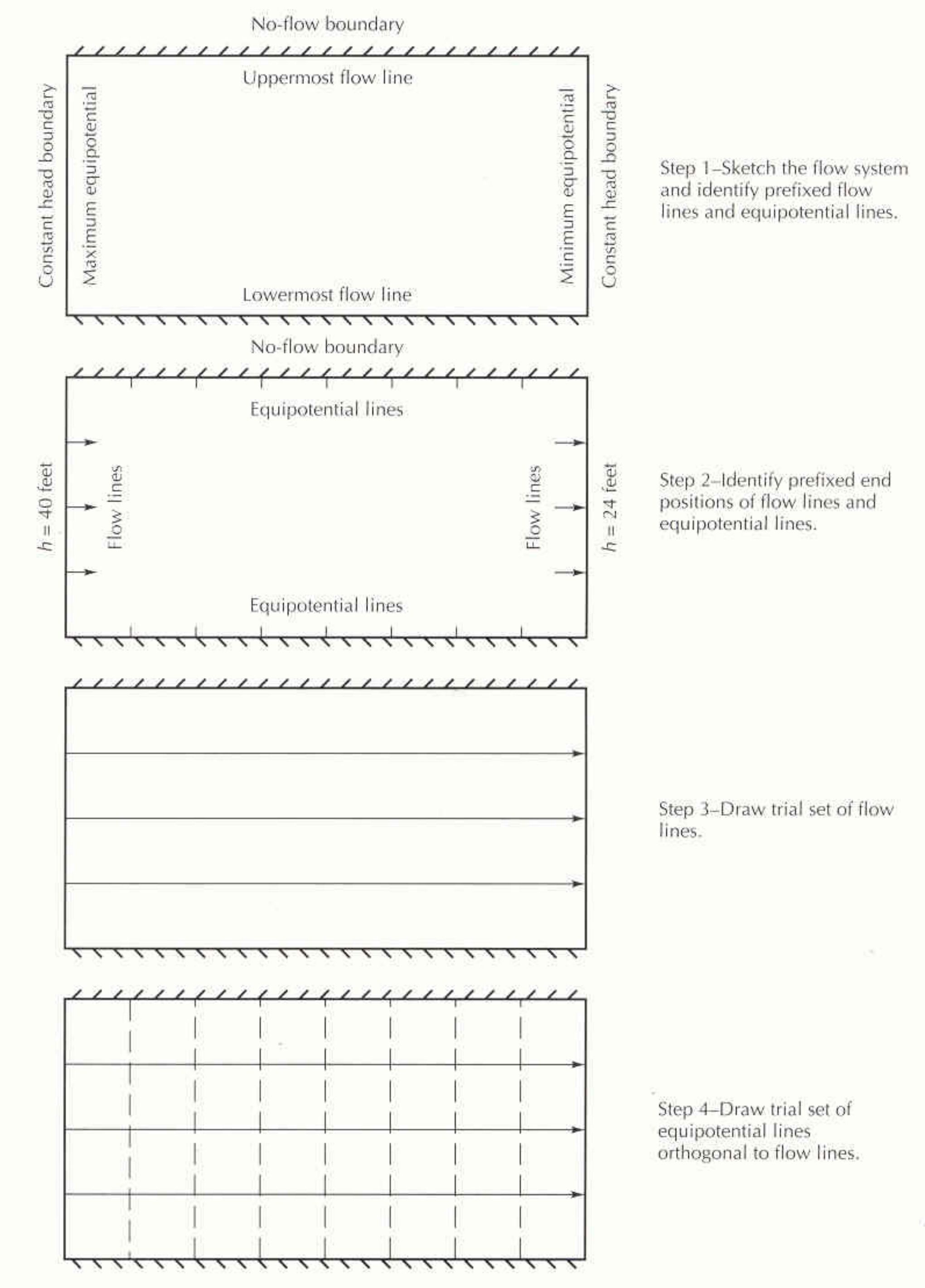


Figure 2.15 Steps in making a flow net

Most beginners at the art of flow-net construction will find an ample supply paper, pencils, and erasers essential. As a check on the quality of the flow net, the diagonals of the squares can be drawn. This should form smooth curves that intersect each other at right angles. This should, of course, be done on a copy of the final product.

In addition to presenting a graphic display of the groundwater flow direction and potential distribution, the completed flow net can be used to determine the quantity of water flowing by the following formula (see **Figure 2.16**)

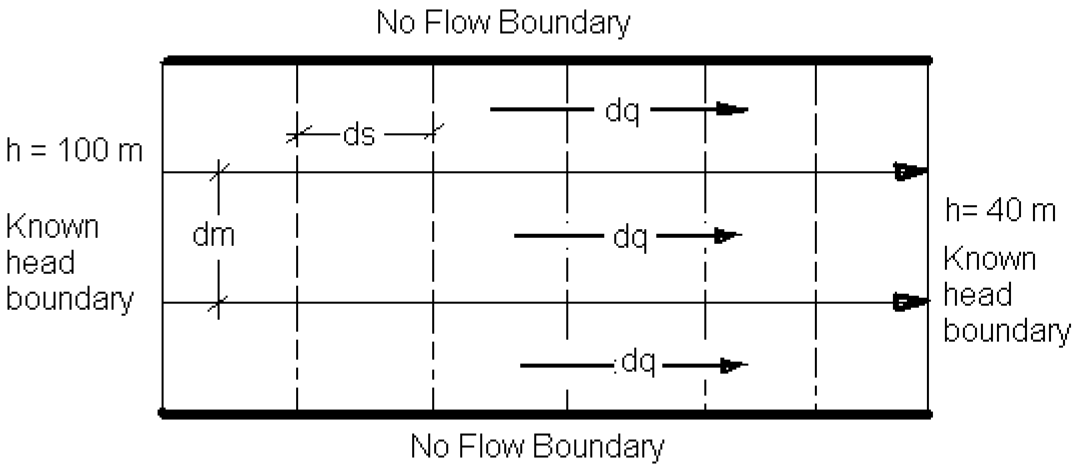


Figure 2.12

- Flow in each stream = dq
- Apply Darcy Law $\Rightarrow dq = K\dfrac{dh}{ds}dm$
- If $dm = ds \Rightarrow dq = K\,dh$ for one stream only.
- For m streams $\Rightarrow$ $q = mK\,dh$ for dh only.
- For total drop across the region (H) with n division of head $H = n\,dh \Rightarrow$ $q = \dfrac{mK\Delta H}{n}$
- The discharge from the entire aquifer =

$$Q = \frac{mK\Delta H}{n} \times width \qquad \textbf{(2.117)}$$

Where,
Q is the total volume discharge.
K is the hydraulic conductivity
m is the number of streamlines
n is the number of equipotential lines

Flow Nets

As we have seen, to work with the groundwater flow equation in any meaningful way, we have to find some kind of a *solution* to the equation. This solution is based on *boundary conditions*, and in the transient case, on *initial conditions*.

Let us look at the two-dimensional, steady-state case. In other words, let the following equation apply:

$$0 = \frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial x}\left(K_y \frac{\partial h}{\partial y}\right) \qquad \text{(Map View)}$$

A solution to this equation requires us to specify boundary condition. For our purposes with flow nets, let us consider

- No-flow boundaries ($\frac{\partial h}{\partial n} = 0$, where *n* is the direction perpendicular to the boundary).
- Constant-head boundaries (*h* = constant)
- Water-table boundary (free surface, *h* is not a constant)

A relatively straightforward graphical technique can be used to find the solution to the GW flow equation for many such situations. This technique involves the construction of a **flow net**.

A flow net is the set of equipotential lines (constant head) and the associated flow lines (lines along which groundwater moves) for a particular set of boundary conditions.

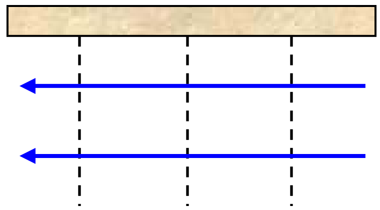
- For a given GW flow equation and a given value of K, the boundary conditions completely determine the solution, and therefore a flow net.

In addition, let us *first* consider only homogeneous, isotropic conditions:

$$0 = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} \qquad \text{(Cross-Section)}$$

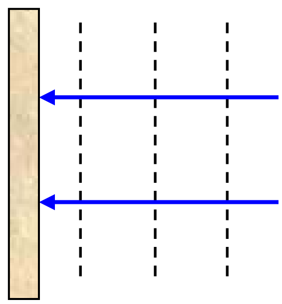
Let's look at flow in the vicinity of each of these boundaries. (Isotropic, homogeneous conditions).

No-Flow Boundaries: $\frac{\partial h}{\partial x} = 0$ or $\frac{\partial h}{\partial y} = 0$ or $\frac{\partial h}{\partial n} = 0$



- Flow is parallel to the boundary.
- Equipotentials are perpendicular to the boundary

Constant-Head Boundaries: h = constant



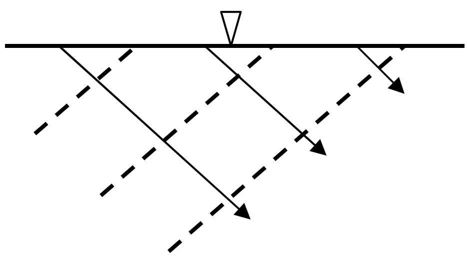
- Flow is perpendicular to the boundary.
- Equipotentials are parallel to the boundary.

Water Table Boundaries: $h=z$

Anywhere in an aquifer, total head is pressure head plus elevation head:

$$h = \psi + z$$

However, at the water table, $\psi = 0$. Therefore, $h = z$

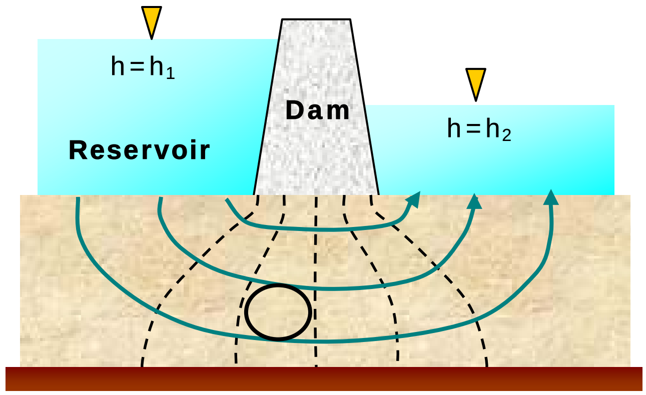


Neither flow nor equipotentials are necessarily perpendicular to the boundary.

Rules for Flow Nets (Isotropic, Homogeneous System):

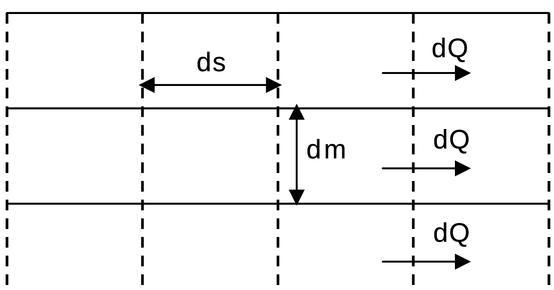
In addition to the boundary conditions the following rules must apply in a flow net:

1) Flow is perpendicular to equipotentials *everywhere*.
2) Flow lines never intersect.
3) The areas between flow lines and equipotentials are "curvilinear squares". In other words, the central dimensions of the "squares" are the same (but the flow lines or equipotentials can curve).
    - If you draw a circle inside the curvilinear square, it is tangential to all four sides at some point.



Why are these circles? It preserves dQ along any stream tube.

dQ = K dm; dh/ds = K dh



If dm $\neq$ ds (i.e. ellipse, not circle), then a constant factor is used.

Other points:
It is not necessary that flow nets have finite boundaries on all sides; regions of flow that extend to infinity in one or more directions are possible (e.g., see the figure above).

A flow net can have "partial" stream tubes along the edge. A flow net can have partial squares at the edges or ends of the flow system.

Calculations from Flow Nets:
It is possible to make many good, quantitative predictions from flow nets. In fact, at one time flow nest were the major tool used for solving the GW flow equation.

Probably the most important calculation is discharge from the system. For a system with one recharge area and one discharge area, we can calculate the discharge with the following expression:

$Q = n_f K\ dH \quad H = n_d\ dH$

Gives: $Q = n_f/n_d\ KH$

Where Q is the volume discharge rate *per unit thickness of section* perpendicular to the flow net; n_f is the number of stream tubes (or flow channels); n_d is the number of head drops; K is the uniform hydraulic conductivity; and H is the total head drop over the region of flow.

- Note that neither n_f nor n_d is necessarily an integer, but it is often helpful if you construct the flow net such that one of them is an integer.
- If you choose n_f as an integer, it is unlikely that n_d will be an integer.
- Note that to do this calculation, you do not need to know any lengths.

Flow Nets in Anisotropic, Homogeneous Systems:

Before construction of a flow net in an anisotropic system ($K_x \neq K_y$ or $K_x \neq K_z$ etc.), we have to *transform* the system.

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = 0$$

For homogeneous K,

$$\frac{\partial^2 h}{\partial x^2} + \frac{K_z}{K_x}\frac{\partial^2 h}{\partial z^2} = 0$$

Introduce the transformed variable

$$Z = \sqrt{\frac{K_x}{K_z}}\, z$$

Applying this variable gives:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial Z^2} = 0$$

With this equation we can apply flownets exactly as we did before. We just have to remember how Z relates to the actual dimension z.

In an anisotropic medium, perform the following steps in constructing a flow net:

1. Transform the system (the area where a flow net is desired) by the following ratio:

$$Z = Z'\sqrt{\frac{K_z}{K_x}}$$

where *z* is the original vertical dimension of the system (on your page, in cm, inches, etc.) and Z' is the transformed vertical dimension.

K_x is the hydraulic conductivity horizontally on your page, and K_z is the hydraulic conductivity vertically on your page. This transformation is not specific to the x-dimension or the y-dimension.

2. On the transformed system, follow the exact same principles for flow nets as outlined for a homogeneous, isotropic system.
3. Perform the inverse transform on the system, i.e.

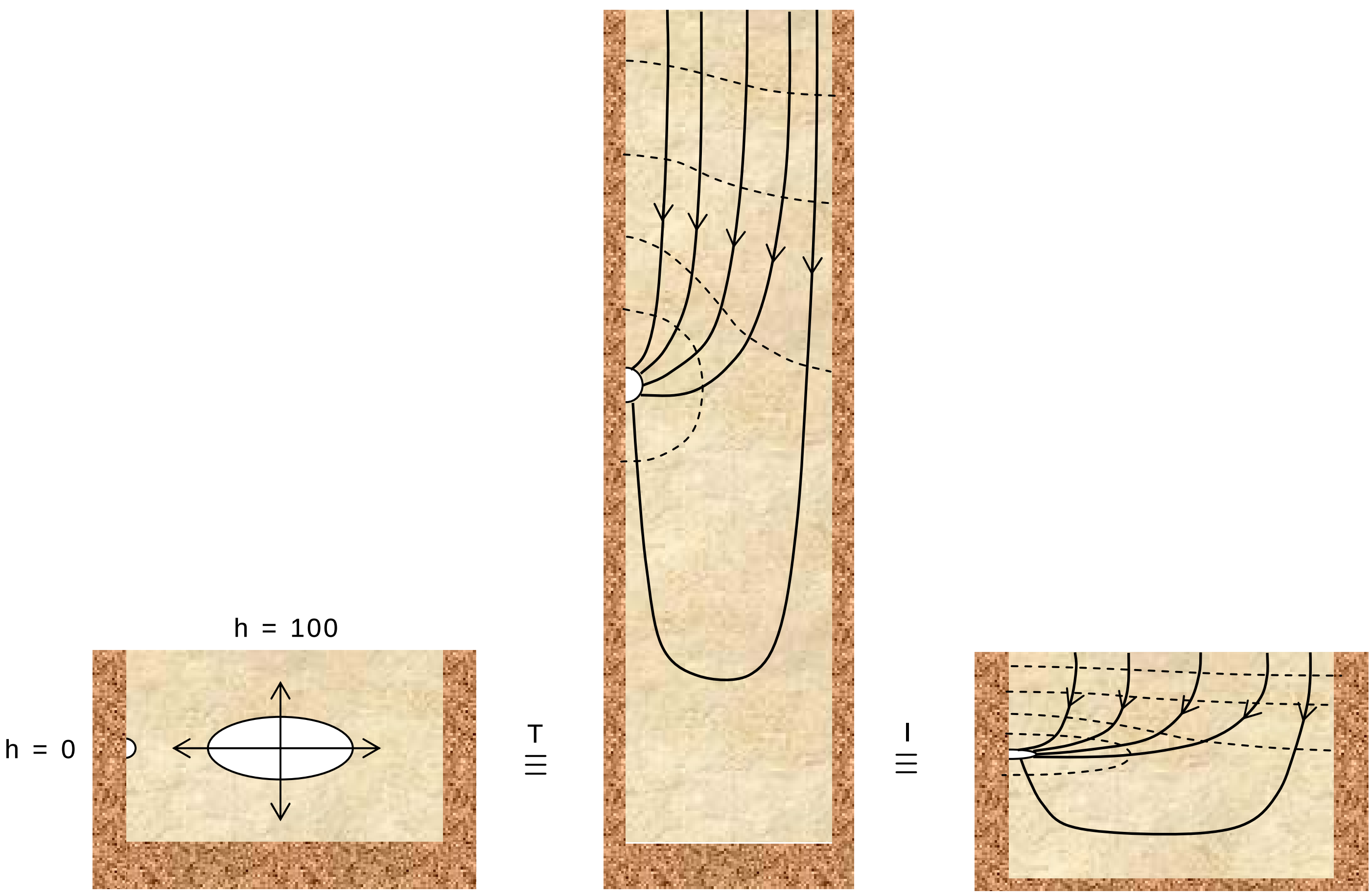
$$Z = Z'\sqrt{\frac{K_z}{K_x}}$$

4. If any flow calculations are needed, do these calculations on the homogeneous (step 2) section. Use the following for hydraulic conductivity:

$$K' = \sqrt{K_x K_z}$$

Where *K'* is the homogeneous hydraulic conductivity of the transformed section. (NOTE: This transformed K' is not real! It is only used for calculations on the transformed section.)

Examples:

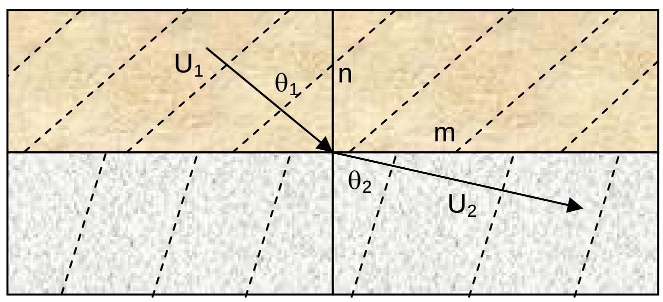


Flow Nets in Heterogeneous Systems:

We will only deal with construction of flow nets in the simplest types of heterogeneous systems. We will restrict ourselves to **layered heterogeneity**.

In a layered system, the same rules apply as in a homogeneous system, with the following important exceptions:

1. Curvilinear squares can only be drawn in ONE layer. In other words, in a two-layer system, you will only have curvilinear squares in one of the layers. Which layer to draw squares in is your choice: in general you should choose the thicker/larger layer.
2. At boundaries between layers, flow lines are refracted (in a similar way to the way light is refracted between two different media). The relationship between the angles in two layers is given by the "tangent law":



$h_1 = h_2 \longrightarrow \frac{\partial h_1}{\partial m} = \frac{\partial h_2}{\partial m}$ (1) No sudden head changes

$K_1 \frac{\partial h_1}{\partial n} = K_2 \frac{\partial h_2}{\partial n}$ (2) Conservation of Mass

	Layer 1	Layer 2
u_x:	$U_1 \sin\theta_1 = -K_1 \frac{\partial h_1}{\partial m}$	$U_2 \sin\theta_2 = -K_2 \frac{\partial h_2}{\partial m}$
u_y:	$U_1 \cos\theta_1 = -K_1 \frac{\partial h_1}{\partial n}$	$U_2 \cos\theta_2 = -K_2 \frac{\partial h_2}{\partial n}$

By (1): $\frac{U_1 \sin\theta_1}{K_1} = \frac{U_2 \sin\theta_2}{K_2}$

By (2): $U_1 \cos\theta_1 = U_2 \cos\theta_2$

$$\frac{K_1}{K_2} = \frac{\tan\theta_1}{\tan\theta_2}$$

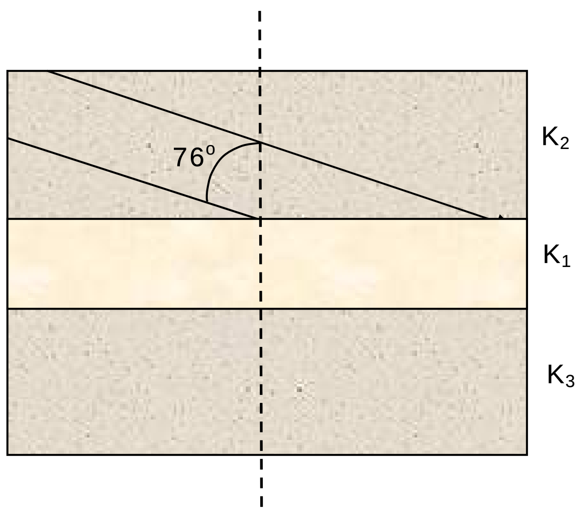
You can rearrange the tangent law in any way to determine one unknown quantity. For example, to determine the angle θ_2:

$$\theta_2 = \tan^{-1}\left(\frac{K_2}{K_1}\tan\theta_1\right)$$

One important consequence for a medium with large contrasts in K: high-K layers will often have almost horizontal flow (in general), while low-K layers will often have almost vertical flow (in general).

Example:

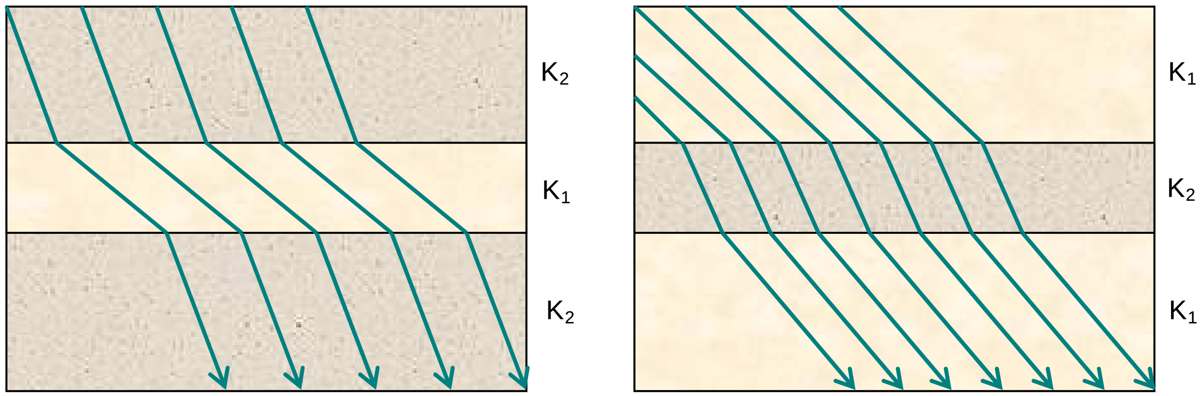
In a three-layer system, $K_1 = 1 \times 10^{-3}$ m/s and $K_2 = 1 \times 10^{-4}$ m/s. $K_3 = K_1$. Flow in the system is 14° below horizontal. What do flow in layers 2 and 3 look like?



$$\theta_2 = \tan^{-1}\left(\frac{1\text{x}10^{-4}}{1\text{x}10^{-3}}\tan 76^\circ\right) = 22^\circ$$

What is the angle in layer 3? If you do the calculation, you will find it is 76° again.

When drawing flow nets with different layers, a very helpful question to ask is “What layer allows water to go from the entrance point to the exit point the easiest?” Or, in other words, “What is the easiest (frictionally speaking) way for water to go from here to there?”



$$\frac{K_1}{K_2} = 10$$